Continuous teleportation of the photon statistics of squeezed states

2005 J. Phys. A: Math. Gen. 38 6571

(http://iopscience.iop.org/0305-4470/38/29/011)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 6571–6577 doi:10.1088/0305-4470/38/29/011

Continuous teleportation of the photon statistics of squeezed states

M Orszag[1] **and D Mundarain**[2]

[1] Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago, Chile
[2] Departamento de Física, Universidad Simón Bolívar, Apartado 89000, Caracas 1080A, Venezuela

Received 20 April 2005, in final form 3 June 2005
Published 6 July 2005
Online at stacks.iop.org/JPhysA/38/6571

Abstract
In this work, we study the oscillations that appear in the photon statistics of a squeezed state in a process that allows teleportation of continuous spectrum variables. In some cases, comparisons are made with the theory of photodetection. The most remarkable result is observed when the fidelity of teleportation is optimized, in that case the teleported statistics is equal to the counting statistics of photoelectrons in non-ideal photocount measurements. We also determine the effect of one-photon subtraction from each arm of the Einstein–Podolsky–Rosen source to enhance the quality of the teleportation process.

PACS numbers: 03.65.−w, 03.65.Ud, 03.67.Mn

1. Introduction

In a quantum teleportation experiment [1, 2] an unknown state is sent by Alice (the sender) to Bob (the receiver) by making use of an entangled Einstein–Podolsky–Rosen (EPR) source, shared by Alice and Bob. Alice makes some measurements in her own sub-system and sends this information to Bob, via a classical channel. In this process, the main feature is the quantum entanglement of the EPR source that allows the collapse of Bob's sub-system due to the measurements done by Alice.

In the simplest case, quantum teleportation is performed on systems with a two-dimensional Hilbert space [1]. For this situation, experimental demonstrations have been performed on single-photon polarized states [3] and teleportation of a polarization state with a complete Bell state measurement [4].

The extension to infinite-dimensional systems was suggested by Vaidman [5] in the context of teleportation of variables with a continuous spectrum, and for this reason we will talk about continuous teleportation. Some specific models were studied later by several authors [6–10].

Here, we are interested in studying how efficiently some relevant physical properties of a quantum state can be teleported, and we will focus on teleportation of the quantities that better characterize the quantum nature of the state under study. In [13, 14], some teleported quadrature fluctuations have been studied. In this work, we will consider the teleportation of the oscillations which appear in the photon statistics of squeezed states. As we know, these oscillations are one of the most impressive manifestations of the quantum nature of squeezed states [15, 16].

This paper is organized as follows. In section 2, we present some basic mathematical concepts used to find in section 3 the teleported statistics of the squeezed states. In section 4, the theory of photodetection is briefly reviewed in order to establish the main result of the paper: when the fidelity of teleportation is optimized, the teleported statistics is equal to the counting statistics of photoelectrons in non-ideal photocount measurements. Section 5 considers the effect of one-photon subtraction from each arm of the EPR source to enhance the quality of the teleportation process. Finally, in section 6 we present our conclusions.

2. The teleportation operator and coherent states

Hofmann *et al* [11] introduced the continuous variable teleportation operator $T_{BA}(g, q, \gamma)$, that allows one to write the output state (the state obtained by Bob in the teleportation process) as

$$|\psi_t(g, q, \gamma)\rangle_B = T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A \tag{1}$$

where $|\psi_{\text{in}}\rangle_A$ is the state that Alice wants to send to Bob. The operator T_{BA} has the following form:

$$T_{BA}(g, q, \gamma) = \left(\frac{1-q^2}{\pi}\right)^{1/2} \sum_{n=0}^{\infty} q^n D_B(g\gamma)|n\rangle_{BA}\langle n|D_A(-\gamma). \tag{2}$$

The indices A and B stand for input (Alice) and output (Bob) states, respectively, which belong to two different Hilbert spaces, g is the gain at Bob's end and q is a parameter related to the degree of squeezing of a two-mode imperfect EPR source, which in this case corresponds to a two-mode squeezed vacuum state

$$|\text{EPR}\rangle = \sqrt{1-q^2}\sum_{n=0}^{\infty} q^n |n, n\rangle_{AB}.$$

The variable γ is a complex number associated with the result of a homodyne measurement done by Alice. The norm of the teleported state,

$$W(\gamma) = {}_B\langle\psi_t(g, q, \gamma)|\psi_t(g, q, \gamma)\rangle_B \tag{3}$$

can be interpreted as the probability density of getting γ in the measurement performed by Alice.

It is easy to show that the teleportation operator acting on a coherent state of Alice's space produces a displaced coherent state in the Bob space,

$$T_{BA}(g, q, \gamma)|\alpha\rangle_A = f_0(g, q, \gamma, \alpha)|g\gamma + q(\alpha - \gamma)\rangle_B \tag{4}$$

where

$$f_0(g, q, \gamma, \alpha) = \left(\frac{1-q^2}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}(1-q^2)|\alpha-\gamma|^2 + \frac{1}{2}(1-gq)(\alpha\gamma^* - \gamma\alpha^*)\right\}. \tag{5}$$

Since the Hermitian conjugate of the teleportation operator $T_{BA}^{\dagger}$ can be written as

$$T_{BA}^{\dagger}(g, q, \gamma) = \left(\frac{1-q^2}{\pi}\right)^{1/2} \sum_{n=0}^{\infty} q^n D_A(\gamma)|n\rangle_{AB}\langle n|D_B(-g\gamma), \tag{6}$$

it is simple to show that

$$T_{BA}^{\dagger}(g, q, \gamma) = T_{AB}(1/g, q, g\gamma). \tag{7}$$

From equations (4) to (7), we easily get

$$T_{BA}^{\dagger}(g, q, \gamma)|\alpha\rangle_B = f_0(1/g, q, g\gamma, \alpha)|\gamma + q(\alpha - g\gamma)\rangle_A \tag{8}$$

$$_B\langle\alpha|T_{BA}(g, q, \gamma) = f_0^*(1/g, q, g\gamma, \alpha)_A\langle\beta + q(\alpha - g\gamma)| \tag{9}$$

Equations (8) and (9) will be useful in section 3 to find the photon statistics of the teleported state.

3. Photon statistics

The average teleported density matrix ρ_t (with the average taken over the γ measurements) is given by

$$\rho_t = \int d^2\gamma\, T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_{AA}\langle\psi_{\text{in}}|T_{BA}^{\dagger}(g, q, \gamma) \tag{10}$$

and therefore the teleported photon statistics is

$$\begin{aligned} P_m^{(t)} &= {}_B\langle m|\rho_t|m\rangle_B \\ &= \int d^2\gamma\, |{}_B\langle m|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A|^2. \end{aligned} \tag{11}$$

In order to evaluate the above expression, we have to compute the matrix element

$$_B\langle m|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A. \tag{12}$$

Consider the coherent state $_B\langle\alpha = x|$ with $x \in \Re$. The inner product with the teleported state $|\psi_t(g, q, \gamma)\rangle_B$ is

$$_B\langle x|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A = e^{-x^2/2}\sum_{n=0}^{\infty}\frac{x^n}{\sqrt{n!}}\,{}_B\langle n|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A. \tag{13}$$

It is not difficult to show that

$$_B\langle m|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A = \frac{1}{\sqrt{m!}}\left\{\left(\frac{\partial}{\partial x}\right)^m\left(e^{x^2/2}{}_B\langle x|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A\right)\right\}_{x=0} \tag{14}$$

where

$$_B\langle x|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A = f_0^*(1/g, q, g\gamma, x)_A\langle\gamma + q(x - g\gamma)|\psi_{\text{in}}\rangle_A. \tag{15}$$

In our particular case, we are interested in the statistics of the teleported state when the input is a squeezed state. For these states one has

$$\begin{aligned} \langle\alpha|\beta, r, \theta\rangle = (\operatorname{sech} r)^{1/2}\exp\big\{&-\tfrac{1}{2}(|\alpha|^2 + |\beta|^2) + \alpha^*\beta\operatorname{sech} r \\ &- \tfrac{1}{2}[e^{i\theta}(\alpha^*)^2 - e^{-i\theta}(\beta)^2]\tanh r\big\}. \end{aligned} \tag{16}$$

Replacing this expression on the right-hand side of equation (15) we can compute the matrix element $_B\langle m|T_{BA}(g, q, \gamma)|\psi_{\text{in}}\rangle_A$ using equation (14). Finally, integrating the square of the matrix element one can obtain the photon statistics that appear in equation (11). Since the analytical expression for the photon statistics is not particularly illustrative, we will only show some numerical results displayed in figures 1 and 2.

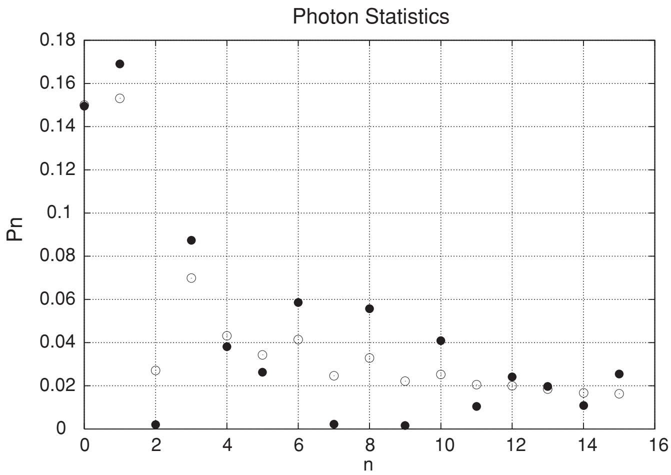

Figure 1. Full circles: squeezed state photon statistics. Open circles: teleported photon statistics. $\alpha = 4$, $r = 2$, $\theta = 0$, and $q = 0.9$, $g = 1$.

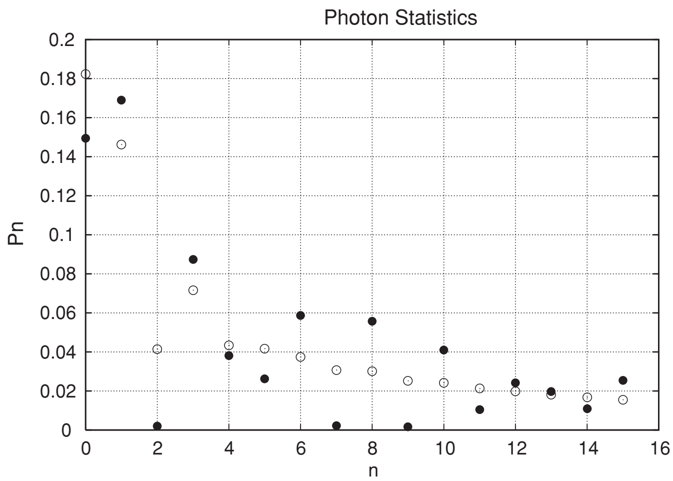

Figure 2. Full circles: squeezed state photon statistics. Open circles: teleported photon statistics. $\alpha = 4$, $r = 2$, $\theta = 0$, and $q = g = 0.9$.

In order to compare the teleported statistics with the photon statistics of the original state sent by Alice, let us recall the photon statistics of squeezed states:

$$P_m = |\langle m|\beta, r, \theta\rangle|^2 = \frac{(\tanh r)^n}{2^n n! \cosh r} \exp\left\{-|\beta|^2 + \frac{1}{2}[\mathrm{e}^{-\mathrm{i}\theta}\beta^2 + \mathrm{e}^{\mathrm{i}\theta}(\beta^*)^2]\tanh r\right\} \times \left|H_n\left(\frac{\beta\,\mathrm{e}^{-\mathrm{i}\theta/2}}{\sqrt{2\cosh r\sinh r}}\right)\right|^2. \tag{17}$$

In figures 1 and 2 we show the photon statistics of a squeezed state P_m and its teleported version $P_m^{(t)}$ for two cases: $q = 0.9$, $g = 1$ (figure 1) and $g = q = 0.9$ (figure 2).

Although the phase space interference, typically represented by the oscillations in the photon statistics of the input state [15], is also present in the teleported version, these

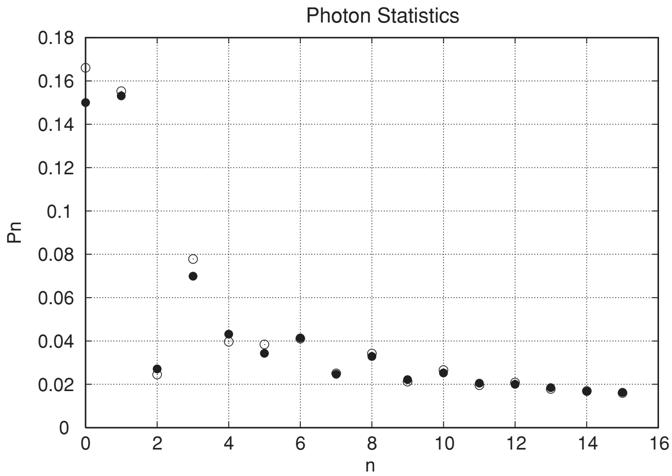

Figure 3. Full circles: teleported photon statistics. Open circles: photocount statistics with efficiency η^{eff}. $\alpha = 4$, $r = 2$, $\theta = 0$, and $q = 0.9$, $g = 1$.

oscillations appear attenuated and can be completely destroyed when the EPR source is poorly correlated or there are no large gains at Bob's end.

4. Photodetection theory

From the photodetection theory, we know that the probability of detecting m photoelectrons, in a measurement with a detector of efficiency $\eta \leqslant 1$, is given by [17]

$$P_m^{(\eta)} = \sum_{n=m}^{\infty} \binom{n}{m} \eta^m (1-\eta)^{n-m} \rho_{nn} \tag{18}$$

where $\rho_{nn} = P_n$ is the original photon statistics of the radiation that is affecting the detector.

It is interesting to note that for $g = q$, the photon statistics of the teleported state $P_m^{(t)}$ is precisely the photocount statistics $P_m^{(\eta)}$ with efficiency $\eta = q^2$. When we plot the photoelectron counting statistics of a squeezed state, equation (18) with $\eta = q^2 = 0.81$, we get exactly the teleported statistics of figure 2. A better grasp of this fact can be obtained by considering the case $g \neq q$. In this case, we cannot find an efficiency that allows us to generate a photoelectron statistics identical to the teleported statistics. However, based on numerical evidence, for values of g that are near q, we found an approximate matching when we use the following expression for the efficiency:

$$\eta^{\mathrm{eff}}(g, q) = gq \exp\left\{\frac{(g-q)^2}{(g+q)^2}\right\}. \tag{19}$$

As we can see in this case, for $g = q$ one obtains $\eta = q^2$ (a perfect matching) and for small differences between g and q we have $0 < \eta \leqslant 1$.

In figure 3 we observe a good agreement between the photoelectron statistics and the photon statistics of the teleported state. In this case we have $g = 1$ and $q = 0.9$.

5. Teleportation with photon subtraction

It has been previously shown that the correlation of the EPR source can be modified by one-photon addition or one-photon subtraction via conditional measurements [12, 13]. This

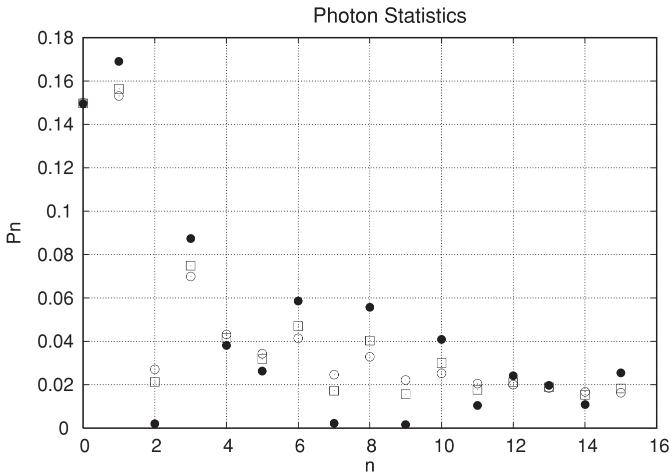

Figure 4. Full circles: squeezed states photon statistics. Open circles: normal teleported photon statistics. Open squares: one-photon subtracted teleported photon statistics. $\alpha = 4$, $r = 2$, $\theta = 0$, and $q = 0.9$, $g = 1$.

can be done experimentally using beam splitters in each arm of the EPR source. Here, we will consider photon subtraction since it is a good choice to increase the EPR correlation, enhancing the teleportation capability of our system.

In this case, the photon statistics is

$$P_n^{(s)} = \int \mathrm{d}^2\beta \left| {}_B\langle n|T_{BA}^{(S)}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A \right|^2, \tag{20}$$

where the corresponding matrix element ${}_B\langle n|T_{BA}^{(S)}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A$ is

$${}_B\langle m|T_{BA}^{(S))}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A = \frac{1}{\sqrt{m!}}\left\{\left(\frac{\partial}{\partial x}\right)^m \left(\mathrm{e}^{x^2/2}{}_B\langle x|T_{BA}^{(S)}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A\right)\right\}_{x=0} \tag{21}$$

and where the teleportation operator with one-photon subtraction is

$$T_{BA}^{(S)}(g,q,\gamma) = \left(\frac{(1-q^2)^3}{\pi(1+q^2)}\right)^{1/2} \sum_{n=0}^{\infty} q^n(n+1)D_B(g\gamma)|n\rangle_{BA}\langle n|D_A(-\gamma). \tag{22}$$

It is simple to see that the above operator can be written in terms of the normal teleportation operator as

$$T_{BA}^{(S)}(g,q,\gamma) = \frac{q(1-q^2)}{\sqrt{1+q^2}}\frac{\partial}{\partial q}T_{BA}(g,q,\gamma) + \frac{1}{\sqrt{1+q^2}}T_{BA}(g,q,\gamma). \tag{23}$$

From this we easily get

$$\begin{aligned} {}_B\langle x|T_{BA}^{(S)}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A &= \frac{q(1-q^2)}{\sqrt{1+q^2}}\frac{\partial}{\partial q}\{{}_B\langle x|T_{BA}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A\} \\ &\quad + \frac{1}{\sqrt{1+q^2}}{}_B\langle x|T_{BA}(g,q,\gamma)|\psi_{\mathrm{in}}\rangle_A. \end{aligned} \tag{24}$$

Then, we can obtain the teleported statistics with one-photon subtraction following the same procedure as in the normal teleportation calculations of section 4. In figure 4 we show the comparison between the photon statistics of a squeezed state, its normally teleported version and the teleported with one-photon subtraction version.

We observe that, for g not too different from q, the photon subtraction reduces the difference between the original and the teleported oscillations, thus showing that the photon subtraction indeed improves the correlation of the EPR source and the whole teleportation process.

6. Conclusions

We have studied the oscillations in the teleported photon statistics of a squeezed state, and compared these with the original oscillations varying various physical parameters related to the correlation of the EPR source and the gain at Bob's end in the teleportation process.

We report the following results:

- In the $q, g \sim 1$ region, we observe a damped oscillation pattern similar to the input one.
- For $q = g$, that is, when the fidelity of the continuous teleportation process is optimized [14], a close comparison with the photodetection theory shows that the teleported statistics and the photoelectron counting statistics with efficiency $\eta = q^2$ are exactly the same. For regions of the parameter space in which $g \sim q$, but not equal, we never have an exact matching. However, from numerical evidence, we nearly reproduce the teleported statistics by taking $\eta^{\text{eff}}(g, q) = gq \exp\left\{\frac{(g-q)^2}{(g+q)^2}\right\}$ in the photoelectron counting statistics.
- Using the fact that the conditional subtraction of one photon from each arm of the EPR source improves its correlation, we calculate the photon statistics for this case, getting an improvement in the oscillations of the teleported photon statistics.

Acknowledgments

DM thanks Professor J Stephany for reading the manuscript and for his useful suggestions. This work was partially supported by DID-USB project GID-030, FONACIT G-2001000712 (DM). It has also been partially supported by FONDECYT (105-1062) and Milenio Project ICMP02-49 (MO).

References

[1] Bennet C H, Brassard G, Crepeau C, Jozsa R, Perez A and Wooters K W 1993 *Phys. Rev. Lett.* **70** 1895
[2] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777
[3] Bouemeester D, Pan J W, Mattle K, Eibi M, Weinfurtes H and Zeilinger A 1997 *Nature* **390** 575
[4] Kim Y, Kulik S P and Shi Y 2001 *Phys. Rev. Lett.* **86** 1370
[5] Vaidman L 1994 *Phys. Rev.* A **49** 1473
[6] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869
[7] Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
[8] Milburn G J and Braunstein S L 1999 *Phys. Rev.* A **60** 937
[9] Jansky J, Koniorczyk M and Gabris V 2001 *Phys. Rev.* A **64** 034302
[10] Ide T, Hofmann H, Kobayashi T and Furusawa A 2001 *Phys. Rev.* A **65** 012313
[11] Hofmann H, Ide T, Kobayashi T and Furusawa A 2000 *Phys. Rev.* A **62** 062304
[12] Opatrny T, Kurizki G and Welsch D G 2000 *Phys. Rev.* A **61** 032302
[13] Maze J R and Orszag M 2004 *J. Opt. B: Quantum Semiclass. Opt.* **6** S566
[14] Maze J R and Orszag M 2004 *J. Mod. Opt.* **51** 2021
[15] Schleich W and Wheeler J A 1987 *Nature* **326** 574
Schleich W and Wheeler J A 1987 *J. Opt. Soc. Am.* B **4** 1715
[16] Mundarain D F and Stephany J 2004 *J. Phys. A: Math. Gen.* **37** 3869
[17] Scully M O and Suhail Zubairy M 1997 *Quantum Optics* (Cambridge: Cambridge University Press)